Thermochimica Acta, 6 (1973) 13–25

THERMODYNAMIC NETWORKS

L. A. MADONNA
Widener College, Chester, Pa. 19013 (U. S. A.)
(Received 13 October 1972)

ABSTRACT

Continuity matrices have been developed for matter, energy and entropy networks. The advantages of this approach are the increased generality and compactness developed by the set of three matrix equations. A simple power plant system or network is analyzed using the matrix method.

NOMENCLATURE

Latin symbols

$\bar{E}$ Power matrix

e Energy $\left(u + \frac{V^2}{2g_c} + \frac{Zg}{g_c}\right)$

e^0 Energy $\left(h + \frac{V^2}{2g_c} + \frac{Zg}{g_c}\right)$

j Generalized flow element in T matrix, with bar column storage matrix
$\bar{J}$ Generalized net flow out matrix (no storage)
m Mass
$\bar{\dot{Q}}$ Heat flux energy matrix
$\bar{S}$ Entropy matrix
$\bar{T}$ Generalized network transport matrix, without bar absolute temperature
$\bar{\dot{W}}$ Work flux (power) matrix
$\bar{I}$ Unit column matrix
h Enthalpy
V Velocity
g_c Gravitational constant, with c = acceleration due to gravity
Z Distance above a selected datum plane

Subscripts

c Column
e Energy
f Final

i Initial
m Mass
n Node or node number

Greek symbol
Δ Difference

INTRODUCTION

Past work in network algebra indicates that balances of matter and charge may be made around *nodal points* in the network. These points are usually electrical, hydraulic or pneumatic junctions. In this presentation and *equipment node* will be considered around which a number of continuity balances will be made. This approach, in the system component sense, is not entirely new and has been considered in the past by almost every engineering discipline. What is new here is the application of classical thermodynamical control volume concepts to multicomponent systems which may be defined as *thermodynamic networks*. In the formulation of the final matrix expressions for continuity the concepts of Lange[1] will be utilized in conjunction with the recent thermodynamics text of Van Wylen and Sonntag. These thermodynamic matrices now present a compact form of solution for large converter networks of many types. A typical solution is shown using the matrix method which demonstrates its simplicity and power.

GENERALIZED CONTINUITY BALANCE AND STORAGE

Continuity matrix

It is possible to show the continuity balance in a simple matrix form. A three-node (three pieces of equipment) system continuity balance will be shown in a generalized form and this will be then extended into a more complex n-node network. If in Fig. 1 a *flow quantity* of some entity is defined as j, then $\overline{T}$ constitutes a form of

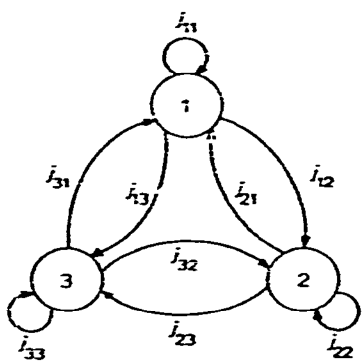

Fig. 1. Three-node thermodynamic network.

connection matrix for the system or network. The j's may be any quantity which may flow into or out of a node.

$$\overline{T} = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} \tag{1}$$

The net flow of any entity out of the node is formed by subtracting the transpose of $\overline{T}$ from $\overline{T}$ and multiplying by a unit column vector, $\overline{I}_c$. The transpose of $\overline{T}$ is:

$$\overline{T}^* = \begin{bmatrix} j_{11} & j_{21} & j_{31} \\ j_{12} & j_{22} & j_{32} \\ j_{13} & j_{23} & j_{33} \end{bmatrix} \tag{2}$$

and thus

$$\overline{T}-\overline{T}^* = \begin{bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33} \end{bmatrix} - \begin{bmatrix} j_{11} & j_{21} & j_{31} \\ j_{12} & j_{22} & j_{32} \\ j_{13} & j_{23} & j_{33} \end{bmatrix} \tag{3}$$

$$\overline{T}-\overline{T}^* = \begin{bmatrix} (j_{11}-j_{11}) & (j_{12}-j_{21}) & (j_{13}-j_{31}) \\ (j_{21}-j_{12}) & (j_{22}-j_{22}) & (j_{23}-j_{32}) \\ (j_{31}-j_{13}) & (j_{32}-j_{23}) & (j_{33}-j_{33}) \end{bmatrix} \tag{4}$$

It is assumed that nodal self-loops do exist then since $j_{11}=j_{11}, j_{22}=j_{22}, j_{33}=j_{33}$. The generalized balance on the three-node network is then

$$\overline{J}_n = \begin{bmatrix} 0 & (j_{12}-j_{21}) & (j_{13}-j_{31}) \\ (j_{21}-j_{12}) & 0 & (j_{23}-j_{32}) \\ (j_{31}-j_{13}) & (j_{32}-j_{23}) & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{5}$$

The expanded form clarifies the matrix expression (5) as seen below

$$\begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} j_{12}-j_{21}+j_{13}-j_{31} \\ j_{21}-j_{12}+j_{23}-j_{32} \\ j_{31}-j_{13}+j_{32}-j_{23} \end{bmatrix} \tag{6}$$

so that the net flow of entity out of node 1 is

$$J_1 = j_{12}+j_{13}-j_{21}-j_{31} \tag{7}$$

for node 2

$$J_2 = j_{21}+j_{23}-j_{12}-j_{32} \tag{8}$$

and for node 3

$$J_3 = j_{31} + j_{32} - j_{13} - j_{23} \tag{9}$$

Equation (5) may be condensed into a simple form which may be defined as a *continuity* matrix.

$$\bar{J}_n = (\bar{T} - \bar{T}^*)\bar{I}_c \tag{10}$$

The number of nodes present is of no consequence so that in the derivation example $n = 3$ but n may be chosen to be of any value.

Storage matrix

A node may store some entity (matter, energy or entropy): the storage may also be indicated by a matrix expression:

Let j_{in} = amount of entity stored in node at start of time (or $t = 0$).
Let j_{fn} = amount of entity stored in node at end of time (or $t = t$).

Hence

$$\Delta j_n = j_{fn} - j_{in} \tag{11}$$

For n nodes then

$$\Delta j_n = \begin{bmatrix} \Delta j_1 \\ \Delta j_2 \\ \Delta j_3 \\ \cdot \\ \cdot \\ \Delta j_n \end{bmatrix} \tag{12}$$

Internal nodes (nodes with both inputs and outputs) may or may not store an entity (mass, energy or entropy). *Boundary nodes* frequently have storage associated with them. A word equation now describes the relationship between expressions (10) and (12)

$$[\text{net flow out of entity}] + [\text{storage of entity}] = 0 \tag{13}$$

or

$$\bar{J}_n + \Delta\bar{j}_n = 0 \tag{14}$$

$$\bar{J}_n = -\Delta\bar{j}_n \tag{15}$$

Equivalently

$$(\bar{T} - \bar{T}^*)\bar{I}_c + \Delta\bar{j}_n = 0 \tag{16}$$

Now recall that the elements in the matrix, $\bar{T}$, may be mass, charge, energy, entropy, vehicles or people. By using the normal thermodynamic laws which can be applied to these nodes a complete analysis of the network may be achieved.

APPLICATION OF THE LAWS OF THERMODYNAMICS TO THE NETWORK

Matter balance

A matter balance for the generalized network now takes the form of the expression shown below if expression (16) is utilized. Let $j = m$ so that

$$(\bar{T}_m - \bar{T}_m^*)\,\bar{I}_c + \Delta m_{nm} = 0 \tag{17}$$

or expanded so as to indicate all elements:

$$\begin{bmatrix} 0 & (m_{12}-m_{21}) & (m_{13}-m_{31}) \ldots & & (m_{1n}-m_{n1}) \\ (m_{21}-m_{12}) & 0 & (m_{23}-m_{32}) \ldots & & (m_{2n}-m_{n2}) \\ (m_{31}-m_{13}) & (m_{32}-m_{23}) & 0 & (m_{34}-m_{43}) & (m_{3n}-m_{n3}) \\ . & . & \text{etc.} & & . \\ . & & & & . \\ (m_{n1}-m_{1n}) & & & & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ . \\ 1_n \end{bmatrix} + \begin{bmatrix} \Delta m_1 \\ \Delta m_2 \\ \Delta m_3 \\ . \\ . \\ \Delta m_n \end{bmatrix} = 0 \tag{18}$$

Equation (18) can be rewritten as*

$$\bar{D}_{mn}\bar{I}_c + \Delta\bar{j}_m = 0 \tag{19}$$

Conservation of energy (first law of thermodynamics)

Matter moving into the network has associated with it a number of typical energies—kinetic, potential flow and work and internal. The most effective manner of writing the energy matrix for a network is to observe eqn (26) and consider energy along with mass elements in the mass matrices. There are two additional terms to consider: one is the heat into or out of each node and the other is the work into or out of each node. Assume, as is normally done, that the *heat in* is *positive* and the *work out* is *positive*. Care must be taken when these matrices are formed that the work and heat signs are correctly inserted.

The first law of thermodynamics normally is written as

$$Q - W = \mathrm{d}E. \tag{20}$$

Using fluxes in eqn (20) gives a power expression

$$\dot{Q} - \dot{W} = \mathrm{d}\dot{E}.$$

In matrix form the balance around each node is simply

$$\bar{\dot{Q}}_c - \bar{\dot{W}}_c = \overline{\mathrm{d}\dot{E}}_c \tag{21}$$

*The node number index "n" will be omitted from here on in the storage term.

where the dE matrix can be written as

$$\overline{\mathrm{d}\dot{E}_c} = \bar{D}_{mn}\bar{I}_c + \Delta\bar{j}_e. \tag{22}$$

Substituting eqn (22) into eqn (21) gives

$$\dot{\bar{Q}}_c - \dot{\bar{W}}_c = \bar{D}_{mn}\bar{I}_c + \Delta\bar{j}_e \tag{23}$$

Equation (20) constitutes an expression which indicates the energy balance around each node in the network. If energy storage in nodes, heat and work flux are known, then all energy node balances can be made with eqn (22); the net flow of energy out of each node is obtainable.

Conservation of entropy (second law of thermodynamics)

The existence of entropic networks may be seen in all levels of process reality. Irreversible losses occur in all actual processes and it may be anticipated that the mass and energy continuity balances should lead to the consideration of an entropic balance on networks.

As in the energy case entropy is associated with matter and its flow throughout the network. This reasoning would then suggest the storage and transport (or flow) matrices as seen in eqns (19) and (22). For the network, the second law of the thermodynamics is written as[2]

$$\mathrm{d}S \geq \dot{Q}/T \tag{24}$$

For every node in the network

$$\overline{\mathrm{d}S} \geq (\dot{Q}/T)_c \tag{25}$$

where T = absolute temperature of node surface (or node in most cases), $\dot{Q}$ = heat flux added or deleted from node and $\overline{\mathrm{d}S}$ may be expressed in entropic storage and flow

$$\overline{\mathrm{d}S} = \bar{D}_{mn}\bar{I}_c + \Delta\bar{j}_s \tag{26}$$

Then

$$\bar{D}_{mn}\bar{I}_c + \Delta\bar{j}_s \geq \dot{\bar{Q}}/T_c \tag{27}$$

Note that the matrix $\Delta\bar{j}_s$ indicates the storage of entropy in all nodes of the system.

Since eqn (24) is written for reversible or irreversible processes a number of cases may be considered for expression (27).

Reversible network case ($Q \neq 0$). In this ideal set of conditions all nodes act in a reversible manner so that

$$\bar{D}_{mn}\bar{I}_c + \Delta\bar{j}_s = \dot{\bar{Q}}/T_c \tag{28}$$

Reversible adiabatic network case ($Q = 0$). All equipment nodes in this approximation are not only reversible processes, but also are adiabatic and hence: $\dot{Q}/T_c = 0$ (no heat fluxes) and $\Delta j_{ns} = 0$ (since changes in entropy storage would mean the transfer of energy or work out of or into interior and boundary nodes). Therefore,

$$\bar{D}_{mn}\bar{I}_c = 0 \tag{29}$$

Irreversible network case ($Q \neq 0$). The network nodes have associated with them irreversible processes so that the entropy of the total network increases thus

$$\bar{D}_{mn}\bar{I}_c + \Delta\bar{j}_s > \dot{Q}/T \tag{30}$$

Net energy flow into (or out of) network

In Fig. 3 the nodes in the cycle have a number of inputs and outputs. Of particular interest here is the inputs and outputs into or out of the region which contains the network.

The energy matrices on the left-hand side of eqn (23) are of interest. Heat input into the network has been taken as positive (output as negative); work energy input has been taken as negative (output as positive). The total net energy either absorbed or rejected by the cycle may be written as

$$E_{Tc} = \sum_i q_i + \sum_i w_i \tag{31}$$

or

$$E_{Tc} = (q_1, q_2, q_3, \dots q_n) \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}_n + (w_1, w_2, w_3, \dots w_n) \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}_n \tag{32}$$

or generally for non-cyclic networks

$$E_{Tc} = \bar{Q}^*\bar{I} + \bar{W}^*\bar{I} \tag{33}$$

and for cyclic networks

$$\bar{Q}^*\bar{I} + \bar{W}^*\bar{I} = 0. \tag{34}$$

Energy network vectors and matrices

Equation (23) has shown the usual energy equation in vector and matrix form. It is then possible to speak of thermodynamic networks as having heat vectors, work vectors, energy flow matrices and energy storage vectors:

$$\begin{array}{ccccccc} \text{heat vector} & & \text{work vector} & & \text{energy flow matrix} & & \text{energy storage matrix} \\ \bar{\dot{Q}} & - & \bar{\dot{W}} & = & \bar{D}_{mn}\bar{I}_c & + & \overline{\Delta\dot{E}}_c \end{array} \qquad (23)$$

The matrix $\bar{D}_{mn}$ is composed of row (or column) vectors which are each related to the flows into and out of all nodes. Also the elements in this matrix are not simple and they themselves are the scalars resulting from the multiplication of energy and unit vectors.

Since generally it was shown that

$\bar{D}_{mn} = \bar{T} - \bar{T}^*$ then for energy

$D_{mn} = \bar{E} - \bar{E}^*$

$$\bar{D}_{mn} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & \ldots & E_{1n} \\ E_{21} & E_{22} & E_{23} & & & \\ E_{31} & \text{etc.} & & & & \\ E_{41} & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ E_{n1} & & & & & \end{bmatrix} - \begin{bmatrix} E_{11} & E_{21} & E_{31} & \cdot & E_{n1} \\ E_{12} & E_{22} & E_{23} & & \\ E_{13} & \text{etc.} & & & \\ E_{14} & & & & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ E_{1n} & & & & \end{bmatrix} \qquad (35)$$

But actually

$$E_{11} = h_{11} + \frac{V_{11}}{2g_c} + Z_{11}\frac{g}{g_c}$$

$$E_{12} = h_{12} + \frac{V_{12}}{2g_c} + Z_{12}\frac{g}{g_c} \qquad (36)$$

so that

$$E_{11} = \left(h_{11}, \frac{V_{11}}{2g_c}, Z_{11}\frac{g}{g_c}\right)\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$E_{12} = \left(h_{12}, \frac{V_{12}}{2g_c}, Z_{12}\frac{g}{g_c}\right)\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad (37)$$

etc.

or

$$\bar{E}_{11} = \bar{e}_{11}{*}\bar{I}_3$$
$$\bar{E}_{12} = \bar{e}_{12}{*}\bar{I}_3 \tag{38}$$

etc.

$$\bar{e}_{11} = \begin{bmatrix} h_{11} \\ \dfrac{V_{11}}{2g_c} \\ Z_1 \dfrac{g}{g_c} \end{bmatrix} = \begin{bmatrix} \text{enthalpic energy} \\ \text{kinetic energy} \\ \text{potential energy} \end{bmatrix}$$

so that in general

$$\bar{D}_{mn} = \begin{bmatrix} (\bar{e}_{11} * \bar{I}_3)\,(\bar{e}_{12} * \bar{I}_3) \ldots \bar{e}_{1n} * \bar{I}_3 \\ (\bar{e}_{21} * \bar{I}_3)\,(\bar{e}_{22} * \bar{I}_3) \ldots \\ \text{etc.} \end{bmatrix} - \begin{bmatrix} (\bar{e}_{11} * \bar{I}_3)\,(\bar{e}_{21} * \bar{I}_3) \ldots (\bar{e}_{1n} * \bar{I}_3) \\ (\bar{e}_{12} * \bar{I}_3)\,(\bar{e}_{22} * \bar{I}_3) \ldots (\bar{e}_{n2} * \bar{I}_3) \\ \text{etc.} \end{bmatrix} \tag{39}$$

Then the $\bar{E}$ matrices may be replaced by $\bar{D}_{mn} = (\bar{Z} - \bar{Z}^*)$ so that

$$\dot{Q} - \dot{W} = (\bar{Z} - \bar{Z}^*)\bar{I} + \overline{\Delta j_c} \tag{40}$$

In eqn (40) it is now understood that three types of energy exist within each element of the Z matrix — enthalpic, kinetic and potential. Equations (23) and (40) are identical except that in eqn (40) the three energy forms are explicitly stated.

Example of use of matrix method: simple power plant network

Energy matrix

An example of the application of this network technique is indicated by using the steam power example given by Van Wylen and Sonntag. It is instructive to compare the approach by these authors and the use of a generalized network mathematics.

The diagram of the steam power plant is indicated in Fig. 2. Figure 3 is a network representation of the same plant. Note that the plant is simple with a cyclic nature quite obvious.

For the system shown in Fig. 2, find:

(a) The heat transfer in line between boiler and turbine, Q_{pI}
(b) Turbine work, w_p
(c) Heat transferred in condenser, q_c
(d) Heat transferred in boiler, q_B

from the given data (from Van Wylen and Sonntag):

$E_{12} = 1315$ Btu/lb $E_{34} = 1045$ Btu/lb $E_{51} = ?$
$E_{23} = 1289$ Btu/lb $E_{45} = 78$ Btu/lb $w_{pu} = 3$ Btu/lb

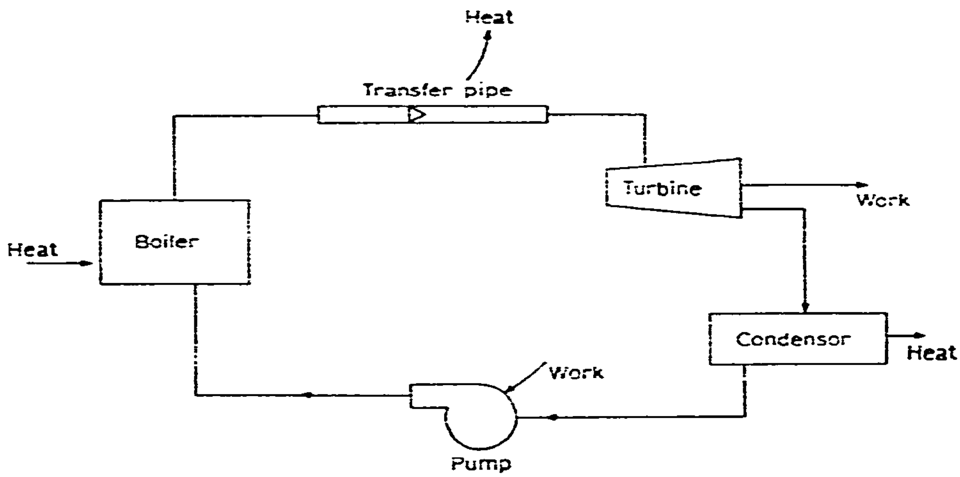

Fig. 2. Steam power plant (schematic).

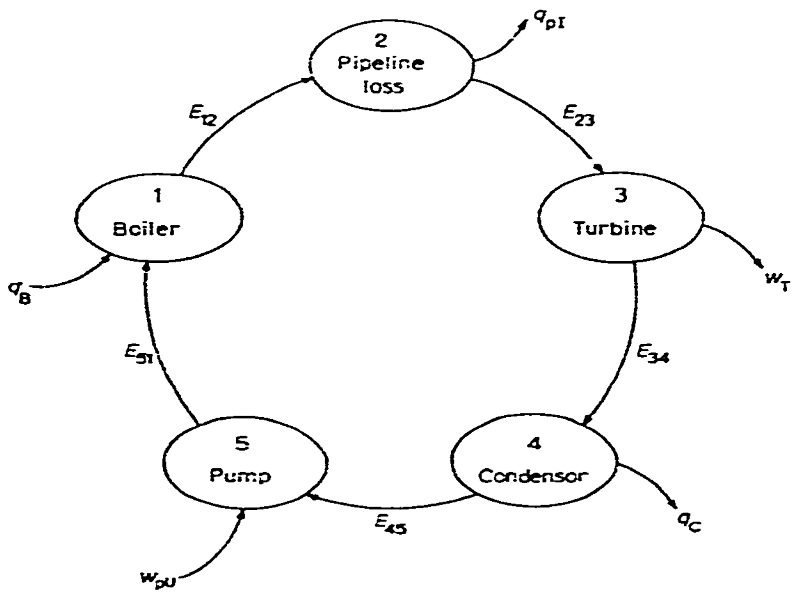

Fig. 3. Steam power plant (network form).

Applying eqn (23) or (40) to this example gives:

$$\begin{bmatrix} +q_B \\ +q_{pI} \\ 0 \\ +q_c \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ w_T \\ 0 \\ +w_{pU} \end{bmatrix} = \left\{ \begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & E_{15} \\ E_{21} & E_{22} & E_{23} & E_{24} & E_{25} \\ E_{31} & E_{32} & E_{33} & E_{34} & E_{35} \\ E_{41} & E_{42} & E_{43} & E_{44} & E_{45} \\ E_{51} & E_{52} & E_{53} & E_{54} & E_{55} \end{bmatrix} - \begin{bmatrix} E_{11} & E_{21} & E_{31} & E_{41} & E_{51} \\ E_{12} & E_{22} & E_{32} & E_{42} & E_{52} \\ E_{13} & E_{23} & E_{33} & E_{43} & E_{53} \\ E_{14} & E_{24} & E_{34} & E_{44} & E_{54} \\ E_{15} & E_{25} & E_{35} & E_{45} & E_{55} \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (41)$$

Since there are no feedbacks on the individual equipment nodes a number of elements in the energy matrix is zero. Note also self-looping is zero (E_{11}, E_{22}, etc. $=0$).

$$\begin{bmatrix} +q_B \\ +q_{pI} \\ 0 \\ +q_c \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ w_T \\ 0 \\ +w_{pU} \end{bmatrix} = \left\{ \begin{bmatrix} 0 & E_{12} & 0 & 0 & 0 \\ 0 & 0 & E_{23} & 0 & 0 \\ 0 & 0 & 0 & E_{34} & 0 \\ 0 & 0 & 0 & 0 & E_{45} \\ E_{51} & 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & E_{51} \\ E_{12} & 0 & 0 & 0 & 0 \\ 0 & E_{23} & 0 & 0 & 0 \\ 0 & 0 & E_{34} & 0 & 0 \\ 0 & 0 & 0 & E_{45} & 0 \end{bmatrix} \right\} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{42}$$

Expression (42) may be written out as:

$$\begin{aligned} +q_B &= E_{12}-E_{51} \\ +q_{pI} &= E_{23}-E_{12} \\ +w_T &= E_{34}-E_{23} \\ +q_c &= E_{45}-E_{34} \\ +w_{pU} &= E_{51}-E_{45} \end{aligned} \tag{43}$$

Total energy requirement for plant

The total net energy requirement for the cycle is gotten by considering eqn (33) and the inputs and outputs; those considered would be only those items from or into the environment.

For the cycle:

$$E_{Tc} = (+q_B, +q_{pI}, 0, +q_c, 0) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + (0, 0, +w_T, 0, +w_{pU}) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{44}$$

$$Q_{Tc} = (q_B - q_{pI} - q_c) + (w_T - w_{pU})$$

For a true cycle $Q_{Tc} = 0$ and therefore

$$(q_B + q_{pI} + q_c) + (w_T + w_{pU}) = 0 \tag{45}$$

Numerical solution

(a) Assumptions (see Van Wylen and Sonntag for their detailed assumptions and solutions).

(1) No heat is lost or gained throughout the system aside from those stated,

(2) No kinetic energy and potential energy changes,

(3) No heat storage, $\overline{\Delta j} = 0$.

Energy equation [see eqn (42) or (43)].

$$\begin{bmatrix} +q_B \\ +q_{pI} \\ 0 \\ +q_c \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ w_T \\ 0 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 & 1315 & 0 & 0 & 0 \\ 0 & 0 & 1289 & 0 & 0 \\ 0 & 0 & 0 & 1045 & 0 \\ 0 & 0 & 0 & 0 & 78 \\ E_{51} & 0 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 & E_{51} \\ 1315 & 0 & 0 & 0 & 0 \\ 0 & 1289 & 0 & 0 & 0 \\ 0 & 0 & 1045 & 0 & 0 \\ 0 & 0 & 0 & 78 & 0 \end{bmatrix} \tag{46}$$

$$\begin{aligned} q_B &= 1315 - E_{51} \\ q_{pI} &= 1289 - 1315 \\ w_T &= 1045 - 1289 \\ q_c &= 78 - 1045 \\ +3 &= E_{51} - 78 \end{aligned} \tag{47}$$

$$\begin{aligned} &\text{(wanted)} \quad q_B = 1315 - E_{51} \\ &\text{(wanted)} \quad q_{pI} = -26 \text{ Btu/lb} \\ &\text{(wanted)} \quad w_T = +244 \text{ Btu/lb} \\ &\text{(wanted)} \quad q_c = -967 \text{ Btu/lb} \\ &\qquad\qquad\quad E_{51} = +81 \text{ Btu/lb} \\ &\qquad\qquad\quad q_B = 1315 - 81 = +1234 \text{ Btu/lb} \end{aligned} \tag{48}$$

Consequently the overall boundary node energy balance [see eqn (44)]

$$E_{Tc} = (q_B, q_{pI}, 0, q_c, 0) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + (0, 0, w_T, 0, w_{pU}) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$E_{Tc} = (1234, -26, 0, -967, 0) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + (0, 0, +244, 0, -3) \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$E_{Tc} = (1234-26-967)+(244-3)$$

$$E_{Tc} = 241-241$$

$$E_{Tc} = 0 \text{ Btu/lb}$$

Since $E_{Tc} = 0$ the cycle is a true thermodynamic cycle.

CONCLUSIONS

Continuity matrices have been developed for energy, matter and entropic networks. The significance in this approach is the generality of the expressions and the amenability to digital computer application to large networks.

REFERENCES

1 O. Lange, *Wholes and Parts*, Pergamon Press, London, 1965.
2 G. J. van Wylen and R. F. Sonntag, *Fundamentals of Classical Thermodynamics*, Wiley, New York, 1965.